Statistics

Fall 2022

Lecture 20

Feb 19-8:47 AM

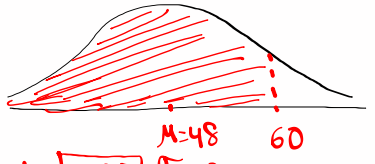
Suppose age of voters are normally dist. with mean of 48 and standard deviation of 10.

$N(48, 10)$

If we randomly select one voter find the prob. that he/she (x) is ($n=1$)

a) below 60 yrs old.

$P(x < 60)$



= normalcdf(−E99, 60, 48, 10) = .885 $\sigma = 10$

b) above 30 yrs old.

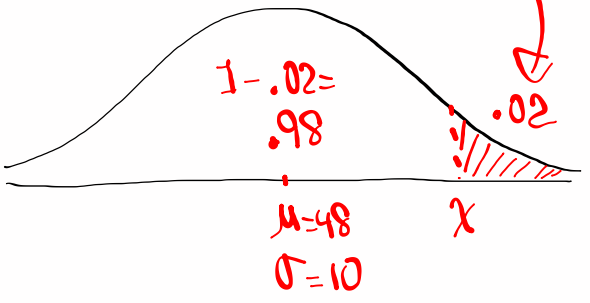
$P(x > 30)$

= normalcdf(30, E99, 48, 10)

= .964

Nov 28-6:02 AM

c) Find an age (x), round to a whole #, that separates the top 2% from the rest.



$x = \text{invNorm}(.98, 48, 10)$

$= 68.538$

≈ 69

Nov 28-6:11 AM

Given $N(160, 20)$

You must use CLT.

For randomly selected groups of 4, find

1) $\mu_{\bar{x}} = \mu = 160$

2) $\sigma^2_{\bar{x}} = \frac{\sigma^2}{n} = \frac{20^2}{4} = \frac{400}{4} = 100$

3) $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{20}{\sqrt{4}} = \frac{20}{2} = 10$

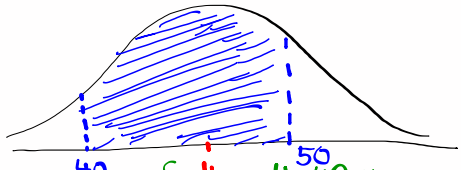
CLT → Central-Limit Theorem

Nov 28-6:14 AM

Suppose ages of all teachers are N.D. with $\mu = 48.5$ Yrs and $\sigma = 7.5$ Yrs. $N(48.5, 7.5)$

If we randomly select groups of 5 ($n=5$) teachers, find the prob. that their mean age ($\bar{x}$) is between 40 and 50 Yrs old.

$P(40 < \bar{x} < 50)$



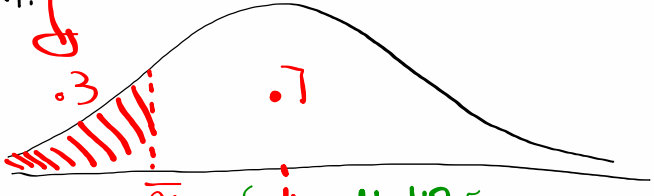
$= \text{normalcdf}(40, 50, 48.5, 7.5/\sqrt{5})$ CLT $\begin{cases} \mu_{\bar{x}} = \mu = 48.5 \\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{7.5}{\sqrt{5}} \end{cases}$

$= .667$

Nov 28-6:19 AM

For randomly selected groups of 4 ($n=4$) teachers, find the mean age ($\bar{x}$) that separates the bottom 30% from the rest.

Round to 1-decimal.



Left Area ↓ μ σ

$\bar{x} = \text{invNorm}(.3, 48.5, 3.75)$

≈ 46.5

CLT $\begin{cases} \mu_{\bar{x}} = \mu = 48.5 \\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{7.5}{\sqrt{4}} = 3.75 \end{cases}$

Nov 28-6:26 AM

Suppose all scores on exam II are normally dist. with mean of 86 and standard deviation of 8. $N(86, 8)$

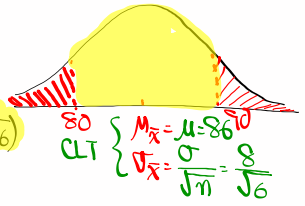
If we randomly select groups of 6 ($n=6$) exams, find the prob. that their mean score ($\bar{x}$) is below 80 or above 90.

$P(\bar{x} < 80 \text{ OR } \bar{x} > 90)$

$= 1 - P(80 < \bar{x} < 90)$

$= 1 - \text{normalcdf}(80, 90, 86, 8/\sqrt{6})$

$= .143 = 14.3\% \approx 14\%$



If we replace OR with and

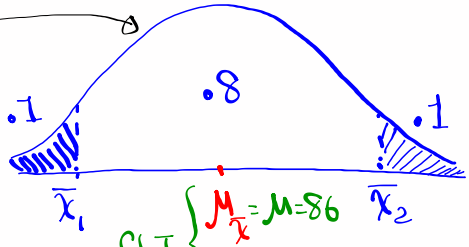
$\Rightarrow$ Answer = 0 $\leftrightarrow$ Mutually exclusive events

Nov 28-6:31 AM

Find two mean scores ($n=4$) for randomly selected groups of 4 exams that separate the middle 80% from the rest. Round to whole #s.

$1 - .8 = .2$

$.2 \div 2 = .1$



CLT $\mu_{\bar{x}} = \mu = 86$, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{8}{\sqrt{4}} = \frac{8}{2} = 4$

$\bar{x}_1 = P_{10} = \text{invNorm}(.1, 86, 4)$

$= 80.874 \approx 81$

Left Area μ σ

$\bar{x}_2 = P_{90} = \text{invNorm}(.9, 86, 4)$

$= 91.126 \approx 91$

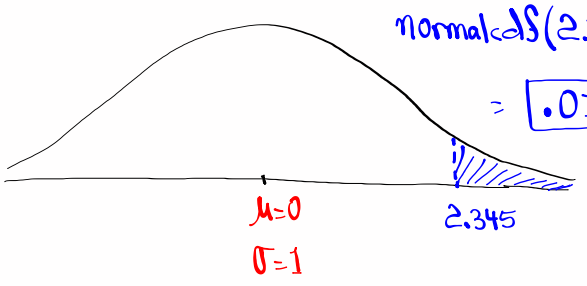
SG 21 & 22 ✓

Nov 28-6:41 AM

Find the area to the right of $Z = 2.345$.

normalcdf(2.345, E99, 0, 1)

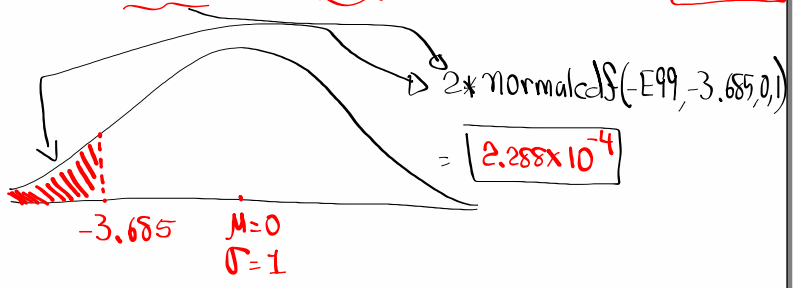
$= .010$



Find twice the area to the left of $Z = -3.685$

$2 * \text{normalcdf}(-E99, -3.685, 0, 1)$

$= 2.288 \times 10^{-4}$

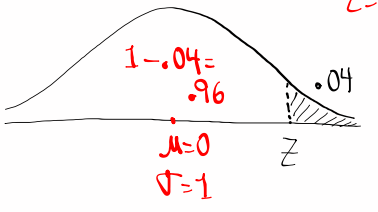


Nov 28-7:07 AM

Find Z using the display below

$Z = \text{invNorm}(.96, 0, 1)$

≈ 1.751



Find two Z-values that separate the middle 94% from the rest.

$1 - .94 = .06$

$.06 \div 2 = .03$

$Z_1 = P_3 = \text{invNorm}(.03, 0, 1) = -1.881$

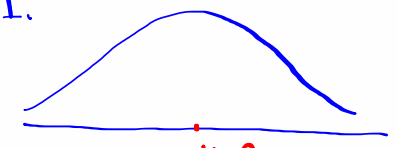
$Z_2 = P_{97} = \text{invNorm}(.97, 0, 1) = 1.881$

Nov 28-7:15 AM

t - Dist

1) Graph is bell-shaped,
Symmetric with total area = 1.

2) $\mu = 0$, σ unknown



3) It comes with degrees of freedom. df

We use tcdf & InvT on this topic.

2nd VARS

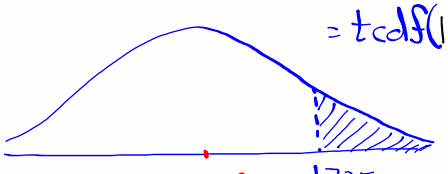
Download recommended Apps in Your Smart Device

Nov 28-7:23 AM

Find $P(t > 1.725)$ with $df = 9$.

= tcdf(1.725, E99, 9)
L U df

= .059

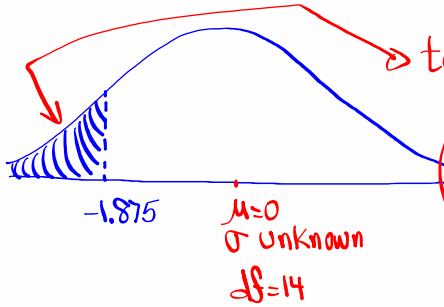


Find $P(t < -1.875)$ with $df = 14$.

2nd [2]

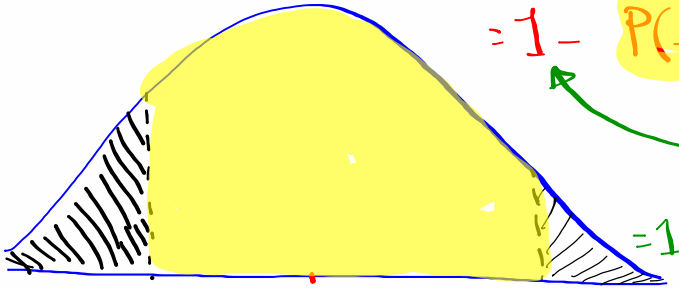
= tcdf(-E99, -1.875, 14)
L U df

= .041



Nov 28-7:28 AM

Find $P(t < -1.5 \text{ OR } t > 1.9)$ with $df = 19$.



$= 1 - P(-1.5 < t < 1.9)$

Total Area = 1

$= 1 - \text{tcdf}(-1.5, 1.9, 19)$

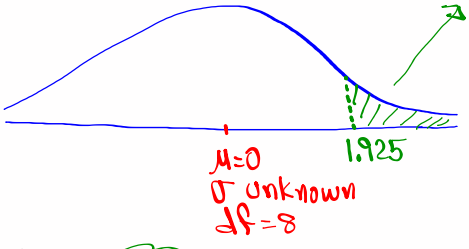
$= .111$

Nov 28-7:35 AM

Find twice the area to the right of $t = 1.925$ with $df = 8$.

$2 * \text{tcdf}(1.925, E99, 8)$

$= .090$



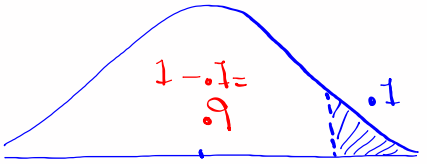
Find twice the area to the left of $t = -1.925$ with $df = 8$.

$2 * \text{tcdf}(-E99, -1.925, 8) = .090$ $df = 8$

Nov 28-7:38 AM

Find the t-value using the display below with df = 11.

$t = invT(.9, 11)$ = 1.363



Left Area, df

Find t using the display below with df = 19.

$t = invT(.2, 19)$ = −.861

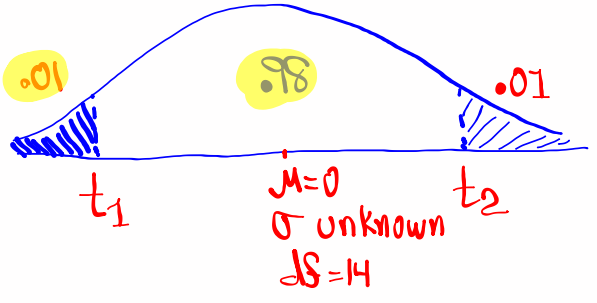
left Area, df

Nov 28-7:46 AM

Find two t-values, Round to 3-decimal places, that separate the middle 98% from the rest with df = 14.

$1 - .98 = .02$

$.02 \div 2 = .01$



$t_1 = P_1 = invT(.01, 14) = -2.624$

$t_2 = P_{99} = invT(.99, 14) = 2.624$

Nov 28-7:53 AM

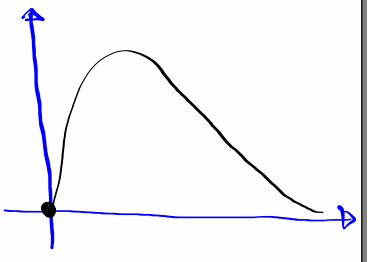
Chi-Square dist.

χ^2

1) Graph begins at 0, and it is skewed to the right

2) Not Symmetric but total area = 1.

3) It comes with df.



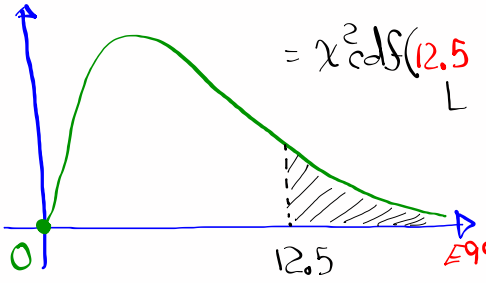
We use χ^2cdf for this topic.

2nd VARS

χ^2cdf(L , U, df)

Nov 28-8:00 AM

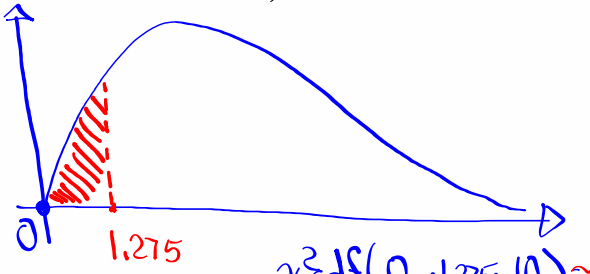
Find $P(\chi^2 > 12.5)$ with df = 8.



= χ^2cdf(12.5, E99, 8)
L U df

= .130

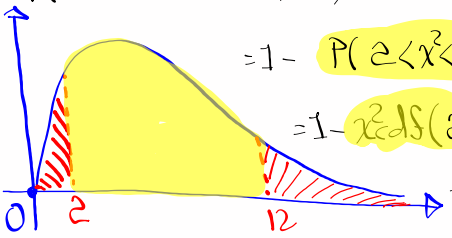
Find $P(\chi^2 < 1.275)$ with df = 10.



χ^2cdf(0, 1.275, 10) ≈ 5.2×10^{-4}
L U df

Nov 28-8:03 AM

Find $P(\chi^2<2 \text{ OR } \chi^2>12)$ with $df=9$.

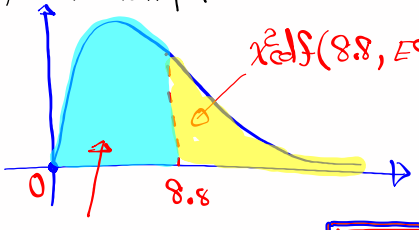


$= 1 - P(2<\chi^2<12)$

$= 1 - \chi^2cdf(2, 12, 9)$

$= .222$

Find the area on each side of $\chi^2=8.8$ with $df=5$, then multiply the smaller area by 2.



$\chi^2cdf(8.8, E99, 5) = .117$

$\chi^2cdf(0, 8.8, 5) = .883$

$2 * \text{Smaller area} = 2 * .117 = .234$

Nov 28-8:11 AM